Introduction to Algebra: Basic Operations and Equations

Introduction and Instructions

Welcome to this introduction to algebra worksheet! Algebra is a branch of mathematics that deals with variables and their relationships. In this worksheet, you will learn about basic algebraic operations and equations. Please read each question carefully and show all your workings and calculations.

Learning Objectives:

- Apply basic algebraic operations to simplify expressions
- Understand the concept of variables and constants in algebraic expressions
- Develop problem-solving skills using algebraic equations

Instructions:

1. Read each question carefully and understand what is being asked.
2. Use a pencil and paper to work out your answers.
3. Show all your workings and calculations for each question.
4. Check your answers to ensure they are reasonable and make sense in the context of the question.

Simplifying Expressions

Simplify the following algebraic expressions by applying basic operations:

1. 2x + 5 + 3x - 2
2. 4y - 2 - 2y + 1
3. 3(2x - 1) + 2(x + 1)
4. 2(x + 3) - 5(x - 2)
5. x + 2 + 2x - 1
6. 3x - 2 + 2x + 1
7. 2(3x - 1) + x + 1
8. x - 2 + 3(x + 1)

Example Solutions

Here are the solutions to the above expressions:

1. 2x + 5 + 3x - 2 = 5x + 3
2. 4y - 2 - 2y + 1 = 2y - 1
3. 3(2x - 1) + 2(x + 1) = 6x - 3 + 2x + 2 = 8x - 1
4. 2(x + 3) - 5(x - 2) = 2x + 6 - 5x + 10 = -3x + 16
5. x + 2 + 2x - 1 = 3x + 1
6. 3x - 2 + 2x + 1 = 5x - 1
7. 2(3x - 1) + x + 1 = 6x - 2 + x + 1 = 7x - 1
8. x - 2 + 3(x + 1) = x - 2 + 3x + 3 = 4x + 1

Solving Equations

Solve the following equations for the variable:

1. 2x + 3 = 7
2. x - 2 = 5
3. 4y = 28
4. x/2 + 2 = 6
5. x + 1 = 9
6. 2x - 3 = 11
7. x/3 + 2 = 5
8. 3x - 2 = 14

Example Solutions

Here are the solutions to the above equations:

1. 2x + 3 = 7 => 2x = 4 => x = 2
2. x - 2 = 5 => x = 7
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6. 2x - 3 = 11 => 2x = 14 => x = 7
7. x/3 + 2 = 5 => x/3 = 3 => x = 9
8. 3x - 2 = 14 => 3x = 16 => x = 16/3

Word Problems

Apply algebraic operations to solve the following word problems:

1. Tom has £15 more than his sister. If his sister has £x, how much money does Tom have in total?
2. A book costs £5 more than a pencil. If the pencil costs £x, how much does the book cost?
3. A bakery sells 250 loaves of bread per day. If they make a profit of £0.50 per loaf, how much profit do they make in a day?
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Example Solutions

Here are the solutions to the above word problems:

1. Tom has £15 more than his sister, so Tom has £x + 15.
2. The book costs £5 more than the pencil, so the book costs £x + 5.
3. The bakery sells 250 loaves of bread per day and makes a profit of £0.50 per loaf, so they make a total profit of 250 x £0.50 = £125 per day.
4. The car travels 250 miles in 5 hours, so it travels 250 / 5 = 50 miles per hour.

Extension Activities

For advanced learners, complete the following challenges:

1. Create and simplify your own algebraic expression using variables, constants, and basic operations.
2. Solve the equation 2x + 5 = 3x - 2 for the variable x.

Example Solutions

Here are the solutions to the above challenges:

1. Example expression: 2x + 3 + 4x - 2 = 6x + 1
2. 2x + 5 = 3x - 2 => 2x - 3x = -2 - 5 => -x = -7 => x = 7

Review and Check

Review your answers and check them for reasonableness. Make sure you have shown all your workings and calculations.

Self-Assessment

Reflect on your learning by asking yourself:

1. What have I learned about basic algebraic operations and equations?
2. What challenges did I face, and how did I overcome them?
3. What could I do to improve my understanding of algebraic concepts?

Additional Resources

For students who require extra support or challenge, the following resources are available:

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Graphing Linear Equations

Graphing linear equations is a fundamental concept in algebra. To graph a linear equation, we need to find the x and y intercepts and plot the points on the coordinate plane. The x-intercept is the point where the line crosses the x-axis, and the y-intercept is the point where the line crosses the y-axis.

Example

Graph the linear equation 2x + 3y = 7. First, find the x-intercept by setting y = 0 and solving for x: 2x + 3(0) = 7 => 2x = 7 => x = 7/2. Then, find the y-intercept by setting x = 0 and solving for y: 2(0) + 3y = 7 => 3y = 7 => y = 7/3. Plot the points (7/2, 0) and (0, 7/3) on the coordinate plane and draw a line through them.

Activity

Graph the following linear equations: x - 2y = 4, 3x + 2y = 5, and 2x - 5y = 1. Find the x and y intercepts and plot the points on the coordinate plane.

Quadratic Equations

Quadratic equations are equations of the form ax^2 + bx + c = 0, where a, b, and c are constants. To solve quadratic equations, we can use factoring, the quadratic formula, or graphing. Factoring involves expressing the quadratic expression as a product of two binomials, while the quadratic formula is a general formula that can be used to solve any quadratic equation.

Case Study

Solve the quadratic equation x^2 + 4x + 4 = 0 using factoring. The equation can be factored as (x + 2)(x + 2) = 0, which gives us two solutions: x + 2 = 0 => x = -2. Therefore, the only solution to the equation is x = -2.

Example

Solve the quadratic equation x^2 - 7x + 12 = 0 using the quadratic formula. The quadratic formula is x = (-b ± √(b^2 - 4ac)) / 2a, where a = 1, b = -7, and c = 12. Plugging these values into the formula, we get x = (7 ± √((-7)^2 - 4(1)(12))) / 2(1) => x = (7 ± √(49 - 48)) / 2 => x = (7 ± √1) / 2 => x = (7 ± 1) / 2. This gives us two solutions: x = (7 + 1) / 2 => x = 4 and x = (7 - 1) / 2 => x = 3.

Functions

A function is a relation between a set of inputs, called the domain, and a set of possible outputs, called the range. Functions can be represented using equations, graphs, or tables. To evaluate a function, we need to substitute the input values into the equation and solve for the output.

Activity

Evaluate the function f(x) = 2x^2 + 3x - 1 at x = 2, x = -1, and x = 0. Substitute the input values into the equation and solve for the output.

Example

Evaluate the function g(x) = x^2 - 2x - 3 at x = 1. Substitute x = 1 into the equation: g(1) = (1)^2 - 2(1) - 3 => g(1) = 1 - 2 - 3 => g(1) = -4.

Systems of Equations

A system of equations is a set of two or more equations that have the same variables. To solve a system of equations, we can use substitution, elimination, or graphing. Substitution involves solving one equation for one variable and substituting that expression into the other equation, while elimination involves adding or subtracting the equations to eliminate one variable.

Case Study

Solve the system of equations x + y = 4 and x - y = 2 using substitution. Solve the first equation for x: x = 4 - y. Substitute this expression into the second equation: (4 - y) - y = 2 => 4 - 2y = 2 => -2y = -2 => y = 1. Then, substitute y = 1 into one of the original equations to find x: x + 1 = 4 => x = 3.

Example

Solve the system of equations 2x + 3y = 7 and x - 2y = -3 using elimination. Multiply the two equations by necessary multiples such that the coefficients of y's in both equations are the same: multiply the first equation by 2 and the second equation by 3. This gives us 4x + 6y = 14 and 3x - 6y = -9. Add both equations to eliminate y: (4x + 6y) + (3x - 6y) = 14 + (-9) => 7x = 5 => x = 5/7. Then, substitute x = 5/7 into one of the original equations to find y: 2(5/7) + 3y = 7 => 10/7 + 3y = 7 => 3y = 7 - 10/7 => 3y = (49 - 10) / 7 => 3y = 39/7 => y = 13/7.

Inequalities

An inequality is a statement that one expression is greater than, less than, greater than or equal to, or less than or equal to another expression. To solve an inequality, we can use similar techniques to solving equations, such as adding, subtracting, multiplying, or dividing both sides by the same value. However, when multiplying or dividing both sides by a negative number, we must reverse the direction of the inequality.

Activity

Solve the inequalities 2x + 3 > 5, x - 2 < 1, and 3x > 12. Use similar techniques to solving equations, and remember to reverse the direction of the inequality when multiplying or dividing both sides by a negative number.

Example

Solve the inequality x - 4 > 2. Add 4 to both sides: x - 4 + 4 > 2 + 4 => x > 6.

Rational Expressions

A rational expression is a fraction of two polynomials. To simplify rational expressions, we can factor the numerator and denominator and cancel out any common factors. To add or subtract rational expressions, we need to find a common denominator.

Case Study

Simplify the rational expression (x^2 + 4x + 4) / (x^2 - 4). Factor the numerator and denominator: ((x + 2)(x + 2)) / ((x + 2)(x - 2)). Cancel out the common factor (x + 2): (x + 2) / (x - 2).

Example

Add the rational expressions (1/x) + (1/(x + 1)). Find a common denominator, which is x(x + 1). Rewrite each fraction with the common denominator: (x + 1)/(x(x + 1)) + x/(x(x + 1)). Add the fractions: (x + 1 + x)/(x(x + 1)) => (2x + 1)/(x(x + 1)).

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