Linear Equations Homework Sheet

Student Name: ______________________

Class: ______________________

Due Date: ______________________

Introduction to Linear Equations

Essential Understanding:

- Definition of linear equations
- X and y intercepts
- Slope and slope-intercept form
- Graphing linear equations

Complete these concept checks:

1. Define and give an example of a linear equation

2. Explain the concept of x and y intercepts

Section 1: Multiple Choice Questions

Choose the correct answer for each question:

1. What is the x-intercept of the equation y = 2x + 3?
   - a) (0, 3)
   - b) (3, 0)
   - c) (-3, 0)
   - d) (1.5, 0)

2. Which of the following equations has a y-intercept of 2?
   - a) y = x + 2
   - b) y = 2x - 1
   - c) y = x - 2
   - d) y = 2x + 1

Section 2: Short Answer Questions

Show your work and explain your reasoning for each question:

1. Find the x-intercept of the equation $y = 3x - 2$

2. Identify the y-intercept of the graph of $y = 2x + 1$

Section 3: Graphing

Graph each equation on the coordinate plane and identify the x and y intercepts:

1. $y = x + 2$

2. $y = 2x - 3$

Section 4: Word Problems

Read each problem carefully and use linear equations to solve:

1. Tom has been saving money for a new bike and has $120 in his savings account. He wants to buy a bike that costs $180. If he saves $12 per week, how many weeks will it take him to have enough money to buy the bike?

2. A company's profit (P) is given by the equation $P = 200x - 1000$, where x is the number of items sold. What is the break-even point (where profit is 0)?

Section 5: Challenge Questions

Complete the following challenges:

1. Find the equation of the line that passes through the points (2, 3) and (4, 5)

2. A company's cost (C) of producing x units of a product is given by the equation $C = 2x + 500$. The revenue (R) from selling x units is given by $R = 10x$. Write an equation for the profit (P) in terms of x

Conclusion

Summary of Key Concepts:

- Linear equations and their key features
- X and y intercepts
- Slope and slope-intercept form
- Graphing linear equations

Final Concept Check:

1. Define and give an example of a linear equation

Advanced Concepts

In this section, we will explore advanced concepts related to linear equations, including systems of linear equations, linear inequalities, and functions. These concepts are crucial in understanding more complex mathematical ideas and have numerous applications in real-world problems.

Example: Solving Systems of Linear Equations

Consider the following system of linear equations: $$\begin{align*} 2x + 3y &= 7 \\ x - 2y &= -3 \end{align*}$$ To solve for x and y, we can use either the substitution method or the elimination method. Let's use the elimination method to find the values of x and y.

Step 1: Multiply the two equations by necessary multiples such that the coefficients of y's in both equations are the same

Multiply the first equation by 2 and the second equation by 3 to make the coefficients of y in both equations equal.

$$\begin{align*} 4x + 6y &= 14 \\ 3x - 6y &= -9 \end{align*}$$

Step 2: Add both equations to eliminate the y variable

Adding both equations gives us: $$\begin{align*} (4x + 6y) + (3x - 6y) &= 14 + (-9) \\ 7x &= 5 \\ x &= \frac{5}{7} \end{align*}$$

Step 3: Substitute the value of x back into one of the original equations to solve for y

Substitute $x = 5/7$ into the first equation: $$\begin{align*} 2\left(\frac{5}{7}\right) + 3y &= 7 \\ \frac{10}{7} + 3y &= 7 \\ 3y &= 7 - \frac{10}{7} \\ 3y &= \frac{49}{7} - \frac{10}{7} \\ 3y &= \frac{39}{7} \\ y &= \frac{13}{7} \end{align*}$$

Case Study: Linear Inequalities

Linear inequalities are used to describe a range of values for a variable. They have many applications in real-world problems, such as determining the range of possible values for a company's profit based on the number of units sold.

Example: Solving Linear Inequalities

Consider the inequality $2x + 5 > 11$. To solve for x, we need to isolate the variable x.

Step 1: Subtract 5 from both sides of the inequality

$2x + 5 - 5 > 11 - 5$

$2x > 6$

Step 2: Divide both sides of the inequality by 2

$2x / 2 > 6 / 2$

$x > 3$

Functions

A function is a relation between a set of inputs, called the domain, and a set of possible outputs, called the range. Functions have many applications in real-world problems, such as modeling population growth or the cost of producing a certain number of units.

Example: Evaluating Functions

Consider the function $f(x) = 2x^2 + 3x - 1$. To evaluate the function at $x = 2$, we substitute $x = 2$ into the function.

Step 1: Substitute x = 2 into the function

$f(2) = 2(2)^2 + 3(2) - 1$

$f(2) = 2(4) + 6 - 1$

Step 2: Simplify the expression

$f(2) = 8 + 6 - 1$

$f(2) = 13$

Case Study: Functions in Real-World Problems

Functions are used to model real-world problems, such as the cost of producing a certain number of units or the population growth of a city.

Example: Modeling Population Growth

Consider a city with an initial population of 100,000 and a growth rate of 2% per year. We can model the population growth using the function $P(t) = 100{,}000(1 + 0.02)^t$, where t is the number of years.

Step 1: Evaluate the function at t = 10

$P(10) = 100{,}000(1 + 0.02)^{10}$

$P(10) = 100{,}000(1.02)^{10}$

Step 2: Simplify the expression

$P(10) = 100{,}000 * 1.21939$

$P(10) = 121{,}939$

Graphing Functions

Graphing functions is an essential tool for visualizing and understanding the behavior of functions. It can help us identify key features, such as the x and y intercepts, and understand how the function changes over time.

Example: Graphing a Linear Function

Consider the function $f(x) = 2x + 1$. To graph the function, we need to find the x and y intercepts and plot the points on the coordinate plane.

Step 1: Find the x-intercept

To find the x-intercept, set $y = 0$ and solve for x.

$0 = 2x + 1$

Step 2: Solve for x

$2x = -1$

$x = -1/2$

Step 3: Find the y-intercept

To find the y-intercept, set $x = 0$ and solve for y.

$y = 2(0) + 1$

Step 4: Solve for y

$y = 1$

(0, 1)

Case Study: Graphing Functions in Real-World Problems

Graphing functions is used to visualize and understand real-world problems, such as the cost of producing a certain number of units or the population growth of a city.

Example: Modeling Cost

Consider a company that produces widgets at a cost of $10 per unit. The company also has a fixed cost of $1000 per month. We can model the cost using the function $C(x) = 10x + 1000$, where x is the number of units produced.

Step 1: Evaluate the function at x = 100

$C(100) = 10(100) + 1000$

$C(100) = 1000 + 1000$

Step 2: Simplify the expression

$C(100) = 2000$

$C(100) = 2000$

Systems of Linear Inequalities

Systems of linear inequalities are used to describe a range of values for two or more variables. They have many applications in real-world problems, such as determining the range of possible values for a company's profit based on the number of units sold.

Example: Solving Systems of Linear Inequalities

Consider the system of linear inequalities: $$\begin{align*} 2x + 3y &\geq 7 \\ x - 2y &\leq -3 \end{align*}$$ To solve the system, we need to graph the two inequalities on the coordinate plane and find the region that satisfies both inequalities.

Step 1: Graph the first inequality

To graph the first inequality, we need to find the x and y intercepts and plot the points on the coordinate plane.

$2x + 3y = 7$

Step 2: Find the x-intercept

To find the x-intercept, set $y = 0$ and solve for x.

$2x + 3(0) = 7$

Step 3: Solve for x

$2x = 7$

$x = 7/2$

Step 4: Find the y-intercept

To find the y-intercept, set $x = 0$ and solve for y.

$2(0) + 3y = 7$

Step 5: Solve for y

$3y = 7$

$y = 7/3$

Case Study: Systems of Linear Inequalities in Real-World Problems

Systems of linear inequalities are used to model real-world problems, such as determining the range of possible values for a company's profit based on the number of units sold.

Example: Modeling Profit

Consider a company that produces two products, A and B. The company's profit is given by the function $P = 2x + 3y$, where x is the number of units of product A sold and y is the number of units of product B sold. The company also has a constraint that $x + 2y \leq 100$. We can model the profit using the system of linear inequalities: \[\begin{align*} 2x + 3y &\geq 100 \\ x + 2y &\leq 100 \end{align*} \]

Step 1: Graph the first inequality

To graph the first inequality, we need to find the x and y intercepts and plot the points on the coordinate plane.

$2x + 3y = 100$

Step 2: Find the x-intercept

To find the x-intercept, set $y = 0$ and solve for x.

$2x + 3(0) = 100$

Step 3: Solve for x

$2x = 100$

$x = 50$

Step 4: Find the y-intercept

To find the y-intercept, set $x = 0$ and solve for y.

$2(0) + 3y = 100$

Step 5: Solve for y

$3y = 100$

$y = 100/3$

Conclusion

In conclusion, linear equations and functions are essential tools for modeling and solving real-world problems. They have many applications in fields such as business, economics, and science. By understanding how to solve linear equations and graph functions, we can make informed decisions and predict outcomes.

Summary of Key Concepts

In this chapter, we covered the following key concepts:

- Linear equations and their applications
- Graphing linear equations
- Systems of linear equations
- Linear inequalities and their applications
- Graphing linear inequalities
- Systems of linear inequalities

Final Thoughts

Linear equations and functions are powerful tools for modeling and solving real-world problems. By mastering these concepts, you will be able to make informed decisions and predict outcomes in a variety of fields.

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Final Concept Check:

1. Define and give an example of a linear equation

Well done on completing your homework children!