

## SECTION 1: DIAGNOSTIC ASSESSMENT

### Conceptual Understanding Diagnostic

#### Learning Objectives:

- Assess prior knowledge of integration concepts
- Evaluate understanding of fundamental integration principles

Complete the following assessment to demonstrate your current understanding of integration techniques.

1. **1.** Define integration in your own words. Consider its geometric and mathematical significance.

2. **2.** Explain the key differences between definite and indefinite integration. Provide specific examples to support your explanation.

3. **3.** Why is the constant of integration (+C) crucial in indefinite integration? Illustrate with a practical example.

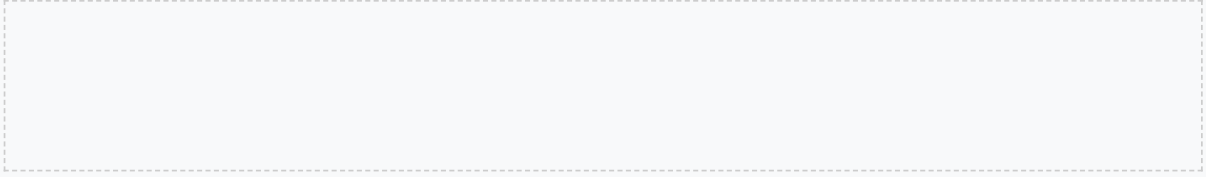
### Quick Calculation Challenge

Solve the following integration problems. Show your complete work.

4. **1.**  $\int(3x^2 + 2x)dx =$

5. **2.**  $\int\sin(x)dx =$

6. 3. Evaluate  $\int_{0 \text{ to } 2} (x^2 + 3) dx =$



## SECTION 2: INTEGRATION TECHNIQUE EXPLORATION

### Power Rule Integration

**Key Technique:** The power rule for integration states that  $\int x^n dx = (x^{(n+1)})/(n+1) + C$ , where  $n \neq -1$

7. 1. Solve the following integration problems:

o  $\int x^4 dx =$

o  $\int (2x^3 - 5x^2 + 3x) dx =$

8. 2. Explain the step-by-step process you used to solve these problems:

### Trigonometric Function Integration

**Integration Identities:**

- $\int \cos(x) dx = \sin(x) + C$
- $\int \sin(x) dx = -\cos(x) + C$
- $\int \sec^2(x) dx = \tan(x) + C$

9. 1. Complete the following integrations:

o  $\int \cos(x) dx =$

o  $\int \tan(x) dx =$

o  $\int \sec^2(x) dx =$

## SECTION 3: ADVANCED INTEGRATION TECHNIQUES

### Substitution Method (u-Substitution)

**Core Principle:** U-substitution allows complex integrals to be simplified by introducing a new variable transformation.

10. **1.** Apply u-substitution to solve the following integrals:

◦  $\int x\sqrt{x^2+1}dx =$

◦  $\int (3x+2)^2 dx =$

11. **2.** Describe the key steps in the u-substitution method:

### Integration by Parts

**Formula:**  $\int u dv = uv - \int v du$


Useful for integrating products of different function types

12. **1.** Solve these integration by parts problems:

◦  $\int x \cos(x) dx =$

◦  $\int \ln(x) dx =$

13. **2.** Create a strategy map for selecting  $u$  and  $dv$  in integration by parts:



## SECTION 4: DEFINITE INTEGRATION APPLICATIONS

### Area Between Curves

**Fundamental Concept:** Definite integrals can calculate the area between two curves by subtracting their integral representations.

14. **1.** Calculate the area between the following curves:

- $f(x) = x^2$  and  $g(x) = 2x$ , from  $x = 0$  to  $x = 2$

- $f(x) = \sin(x)$  and  $g(x) = \cos(x)$ , from  $x = 0$  to  $\pi/4$

15. **2.** Explain the step-by-step process for finding area between curves:

### Volume of Revolution

**Disk Method:**  $V = \pi \int [a \text{ to } b] [f(x)]^2 dx$

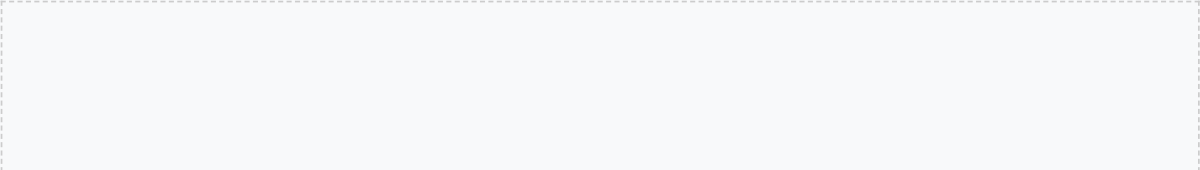
Calculates volume by rotating a curve around an axis

16. **1.** Compute volumes of revolution:

- Rotate  $y = x^2$  around x-axis from  $x = 0$  to  $x = 2$

- Rotate  $y = \sqrt{x}$  around y-axis from  $y = 0$  to  $y = 4$

17. 2. Diagram and explain the disk method visualization:





## SECTION 5: ADVANCED INTEGRATION CHALLENGES

### Improper Integrals

**Definition:** Integrals with infinite limits or discontinuous functions

Requires limit evaluation and careful analysis

18. **1.** Evaluate these improper integrals:

◦  $\int[1 \text{ to } \infty] (1/x^2)dx =$

◦  $\int[0 \text{ to } 1] (1/\sqrt{x})dx =$

19. **2.** Discuss convergence and divergence criteria:

### Numerical Integration Techniques

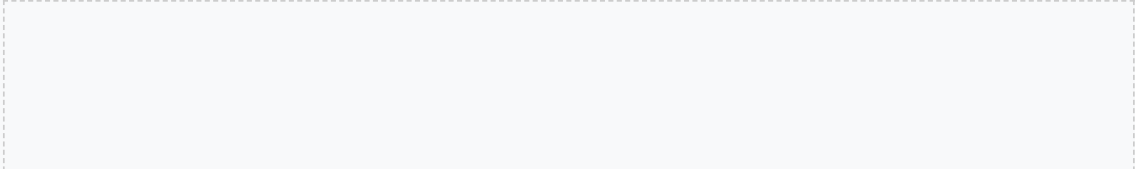
**Methods:**

- Trapezoidal Rule
- Simpson's Rule
- Numerical approximation strategies

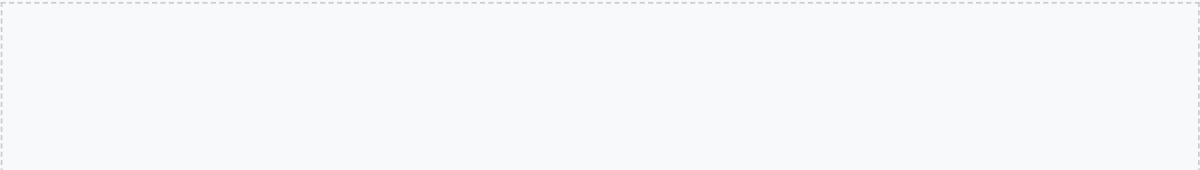
20. **1.** Apply numerical integration techniques:

◦ Approximate  $\int[0 \text{ to } 1] \sin(x)dx$  using Trapezoidal Rule with 4 subintervals

◦ Compare numerical and analytical integration results



21. **2.** Create a comparison chart of numerical integration methods:



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◦  $\int \sec^2(x) dx =$

### Final Reflection

Take a moment to reflect on the integration techniques you've explored. What new insights have you gained about calculus and mathematical problem-solving?